

Part I

Appendices

Appendix A

A primer in Fourier transform using MATLAB

Appendix B

Hologram preparation for spatial light modulator

We are able to produce desired target patterns in the far-field spatial correlations of the twin beam by using a structured pump beam approach. The structured pump beam has a specific distribution of angular spectrum in accordance with the convolution function Eq. (14) which is itself fixed by target pattern selected. The desired angular spectrum in the pump field can be achieved via the application of a suitable phase pattern ϕ at the spatial light modulator (SLM) which is transferred to the pump field upon reflection. The reflected beam then goes through a $4f$ -imaging system such the field distribution with the phase pattern is mapped to the Rb cell center (pump beam waist location). The main goal of the hologram preparation is then to calculate the phase, $\phi(\rho_1, \rho_2)$, for the pump electric field such that E_{out} in the far-field matches with the target field pattern T (see Fig. (5)), i.e.,

$$\begin{aligned} E_{\text{out}} &= \mathcal{E}_o \left(\frac{f\mathbf{k}_p^\perp}{k} \right) \star \mathcal{E}_o \left(\frac{f\mathbf{k}_p^\perp}{k} \right) \\ &= \mathcal{F} \left\{ \left[E_o(\boldsymbol{\rho}) e^{i\phi(\boldsymbol{\rho})} \right]^2 \right\} = T \end{aligned} \tag{B.1}$$

where, $\mathbf{k}_p^\perp$ is the pump transverse momentum vector and $\boldsymbol{\rho} = (\rho_1, \rho_2)$ is the coordinate in transverse plane at the cell center.

After calculating the value of E_{out} for an initial phase, subsequent better values for ϕ can be realized by assigning a cost function to any deviations from the target pattern T and then using a minimization algorithm to reduce the cost. A low cost value gives high degree of match between E_{out} and T and ensures an optimal angular spectrum for the pump field is present at the non-linear medium. For the optimization of E_{out} , we use a conjugate minimization algorithm (Bowman et al., 2017) coupled with the mixed-region-amplitude-freedom (MRAF) approach. In MRAF, the transverse plane of target field is divided into a signal and a noise region and the cost function is only evaluated over the signal region allowing the E_{out} to take any values outside this region. Therefore, a mismatch between T and E_{out} values in the noise region doesn't affect the cost values. For the minimization of the cost function (C) using conjugate gradient minimization algorithm, we define

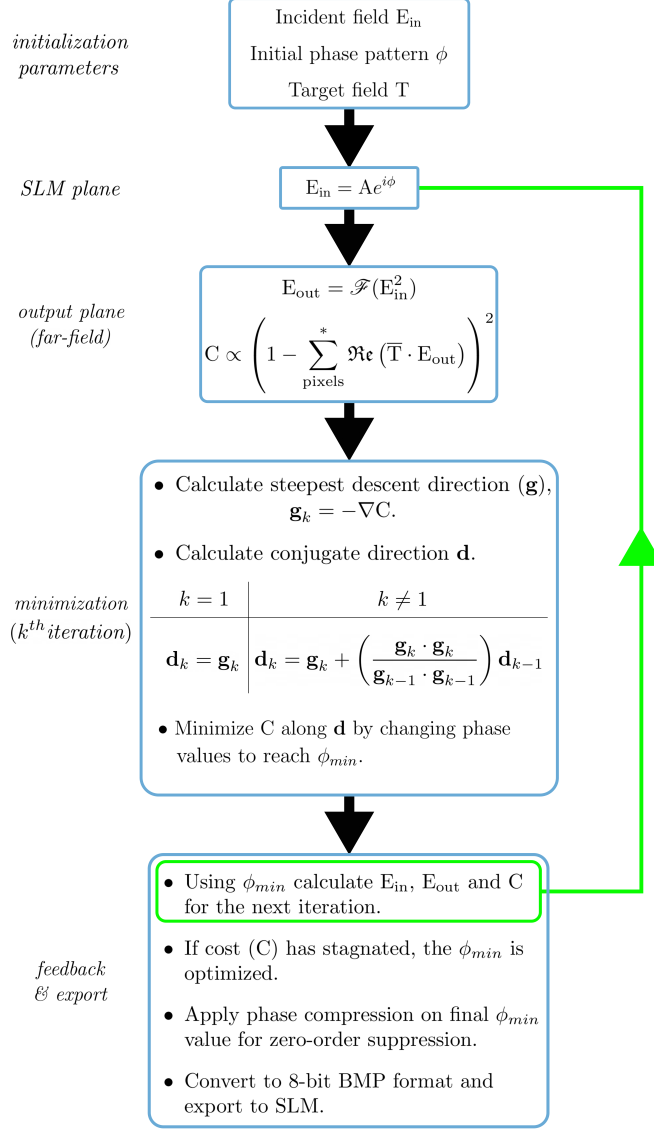


Figure B.1: The flowchart for calculating optimal E_{out} using the conjugate minimization approach coupled with MRAF algorithm. The pump beam is modified with the phase pattern at the SLM plane such that E_{out} at the far-field plane matches the desired target field T . Beginning with an initial guess phase ϕ and the corresponding E_{out} is compared with Target pattern T to estimate an initial cost using the cost function. This cost function is then iteratively minimized along various conjugate directions ($\mathbf{d}$) until the cost function stagnates. After an additional phase-compression to eliminate the zero order in SLM reflection, the final phase pattern is converted into bit-map and is applied on the SLM.

$$C = 10^d \left(1 - \sum_{\text{pixels}}^* \Re(\bar{T} \cdot E_{out})\right)^2 \quad (\text{B.2})$$

where, $d = 10$, $\bar{T}$ represents complex conjugate of the target field, and $(\cdot)$ operation denotes point-wise multiplication. Following the MRAF approach, the cost function is evaluated over the signal region only as indicated by an asterisk over the summation. Depending on the spatial resolution and/or size of the grid over which phase ϕ is defined, the cost function represents a surface in N^2 -dimensional space for a $N \times N$ grid size. We choose a 512×512 grid size to minimize C optimizing by 512^2 independent phase values. To achieve this, gradient of the cost function, $\partial C / \partial \phi$, is calculated on the multi-dimensional cost surface based on which a conjugate direction is chosen, see Fig. (5). While descending along the conjugate direction, the cost function is then reduced in finite size steps until a minimum is reached. Final phase value(s) $\phi_{\min}$ is then used to calculate a new gradient and a corresponding conjugate direction for further minimization. This process is iterated till the cost function stagnates. A large constant (10d) in the cost function provides faster convergence while avoiding local minima. The $\phi_{\min}$ is then further phase compressed for the zero-order suppression (Liang, J., Wu, S.-Y., Fatemi, F. K. & Becker, M. F. 2012). This is due to a limited efficiency of the SLM as part of the field doesn't go through the required phase changes and is reflected without any change (and presents itself as a zero-order in the Fourier plane of the SLM). The phase values in the rest of the field can be adjusted to suppress contribution from the unchanged field. In the end, after the compression, the final phase value is converted to 8-bit format hologram and is exported to SLM. As mentioned in the main text, a phase-only hologram doesn't significantly reduce pump intensity and therefore provides a higher gain for the FWM process in our experiment. In addition to this, a spatially amplitude-modulated pump would amount to a substantially higher pump background in both the spatial and temporal detection.